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Bergshoeff, Eric; Proeyen, Antoine Van

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Symmetries of string, M- and F-theories

Eric Bergshoeff¹ and Antoine Van Proeyen^{2,3}

¹ Institute for Theoretical Physics, Nijenborgh 4, 9747 AG Groningen, The Netherlands

² Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium

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Abstract

The $d = 10$ type II string theories, $d = 11$ M-theory and $d = 12$ F-theory have the same symmetry group. It can be viewed either as a subgroup of a conformal group $OSp(1|64)$ or as a contraction of $OSp(1|32)$. The theories are related by different identifications of their symmetry operators as generators of $OSp(1|32)$. T- and S-dualities are recognized as redefinitions of generators. Some (s, t) signatures of spacetime allow reality conditions on the generators. All of those that allow a real structure are related again by redefinitions within the algebra, due to the fact that the algebra $OSp(1|32)$ has only one real realization. The redefinitions include space/space, time/time and space/time dualities. A further distinction between the theories is made by the identification of the translation generator. This distinguishes various versions of type II string theories, in particular the so-called $*$ -theories, characterized by the fact that the P_0 generator is not the (unique) positive-definite energy operator in the algebra.

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1. Introduction

The group $OSp(1|32)$ was already mentioned in the first papers on $d = 11$ supergravity [1]. This algebra and its extension $OSp(1|64)$ appeared as anti-de Sitter (adS) and superconformal algebras in $d = 10$ and 11 Minkowski theories [2] long ago, and received new attention related to the M-theory algebra [3]. The adS or conformal algebras received new attention in a recent paper on the superconformal aspects of $d = 11$ theories [4] and in two-time theories [5, 6]. In these two cases, the $OSp(1|64)$ conformal group appeared. In the physical theories that we consider, we need the subgroup of $OSp(1|64)$ that is a contraction of $OSp(1|32)$ in a way that will be clarified below.

Our initial motivation to study the role of the $OSp(1|32)$ algebra was related to Euclidean theories. When one considers the D -instanton [7], one often considers the bosonic theory,

³ Onderzoeksdirecteur FWO, Belgium.

ignoring its possible embedding in the supersymmetric theory. In particular, one makes use of the IIB theory in Euclidean space, while the latter cannot be formulated as a supersymmetric theory with real fields, as we will show below. Note that the connection between these Euclidean theories and the Minkowski string theories involve a duality between theories of different spacetime signature [8].

A second question that was posed when we started this research, was related to the observation that in many super-Euclidean theories one makes use of complexification of the fields and in other cases one does not [9]. We would like to know when it is necessary to do so, and when it can be avoided.

Apart from the possibility of no time directions, one is also interested in theories with more time directions [5, 6, 10–13]. Therefore, it looked natural to extend our investigation to an arbitrary spacetime signature.

This leads to a web of dualities between theories in $d = 10, 11$ and 12 of different spacetime signature, similar to what has been found in [11]. We obtain these dualities from an algebraic approach, which puts the contraction of $OSp(1|32)$ as a unifying principle. The different theories are then just many faces of the same underlying symmetry group. This paper summarizes the results obtained in [14].

In section 2, we clarify the relation between the super-Poincaré algebra that we consider here and the full $OSp(1|32)$ as a super-adS algebra or $OSp(1|64)$ as a superconformal algebra. Then we go to the main three steps of our results. In section 3 we consider the complex algebra and its realizations in the different dimensions, in section 4 we discuss the real algebra and its realizations in different spacetime signatures, and in section 5 we identify the translation operator, distinguishing between different Lagrangian theories for the same spacetime signature. Throughout the work we indicate the dualities connecting all the theories. Finally, a short summary is given in section 6. Some extra figures are given in [15].

2. Poincaré algebras as contractions of anti-de Sitter and as subalgebras of conformal algebras

The Poincaré algebra contains translations and the Lorentz algebra.

$$\begin{aligned} [P_\mu, P_\nu] &= 0, & [P_\mu, M_{\nu\rho}] &= \eta_{\mu[\nu} P_{\rho]}, \\ [M_{\mu\nu}, M_{\rho\sigma}] &= \eta_{\mu[\rho} M_{\sigma]\nu} - \eta_{\nu[\rho} M_{\sigma]\mu}. \end{aligned} \quad (2.1)$$

It is a semi-direct product of $SO(d-1, 1)$ with translations.

In the adS algebra, the translations unify with the rotations to $SO(d-1, 2)$. The translations no longer commute,

$$[P_\mu, P_\nu] = \frac{1}{2R^2} M_{\mu\nu}, \quad (2.2)$$

where a parameter R appears that is the radius of the adS space. The Poincaré algebra is the contraction of the algebra (2.2) obtained by $R \rightarrow \infty$. If the right-hand side of (2.2) were $-\frac{1}{2R^2} M_{\mu\nu}$, we would have the de Sitter algebra, rather than the anti-de Sitter algebra. The structure of the algebra is clarified by defining

$$M_{d\mu} \equiv -M_{\mu d} \equiv R P_\mu, \quad (2.3)$$

to obtain the algebra

$$[M_{\hat{\mu}\hat{\nu}}, M_{\hat{\rho}\hat{\sigma}}] = \eta_{\hat{\mu}[\hat{\rho}} M_{\hat{\sigma}]\hat{\nu}} - \eta_{\hat{\nu}[\hat{\rho}} M_{\hat{\sigma}]\hat{\mu}}, \quad (2.4)$$

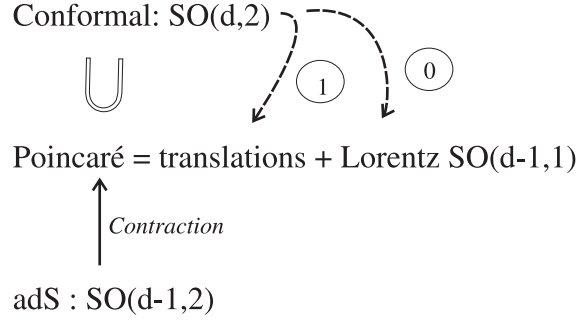


Figure 1. Schematic structure of bosonic algebras. The numbers in circles denote the weights discussed in the text.

where $\hat{\mu} = 0, \dots, d$, and $\eta_{\hat{\mu}\hat{\nu}} = \text{diag}(- + \dots + -)$ (for de Sitter, rather than adS, the latter $-$ would be another $+$).

The conformal algebra is $SO(d, 2)$ having the Poincaré algebra as a subalgebra (the first two lines of these equations):

$$\begin{aligned}
 [P_\mu, P_\nu] &= 0, & [P_\mu, M_{\nu\rho}] &= \eta_{\mu[\nu} P_{\rho]}, \\
 [M_{\mu\nu}, M_{\rho\sigma}] &= \eta_{\mu[\rho} M_{\sigma]\nu} - \eta_{\nu[\rho} M_{\sigma]\mu}, \\
 [K_\mu, M_{\nu\rho}] &= \eta_{\mu[\nu} K_{\rho]}, & [K_\mu, K_\nu] &= 0, \\
 [P_\mu, K_\nu] &= 2(\eta_{\mu\nu} D + 2M_{\mu\nu}), \\
 [D, P_\mu] &= P_\mu, & [D, M_{\mu\nu}] &= 0, & [D, K_\mu] &= -K_\mu.
 \end{aligned} \tag{2.5}$$

The commutation relations with the dilations in the last line define a weight for all the generators, giving a weight 1 to P and weight 0 to $M_{\mu\nu}$. All the commutation relations are consistent with the weight assignments. In this way the algebra is visually represented by the diagram

$$\begin{aligned}
 1 : & \quad P_\mu \\
 0 : & \quad D, M_{\mu\nu} \\
 -1 : & \quad K_\mu.
 \end{aligned} \tag{2.6}$$

This is related to the three-graded structure, which for the superalgebras will be the five-graded structure, that Murat Günaydin mentioned at this conference (see his contribution to the proceedings).

Schematically, the structure of the algebras that we have encountered contains a contraction of an adS algebra, and an inclusion of the Poincaré algebra in a conformal algebra (see figure 1). Note that the Poincaré algebra is built from the weight 1 generators and the Lorentz generators in the weight 0 part of the conformal algebra. The weight -1 generators are realized nonlinearly in the physical theories.

For the superalgebras (we take four dimensions and $N = 1$), the schematic picture looks similar (see figure 2). The adS algebra that is at the basis, is $OSp(1|4)$. Rescaling the P

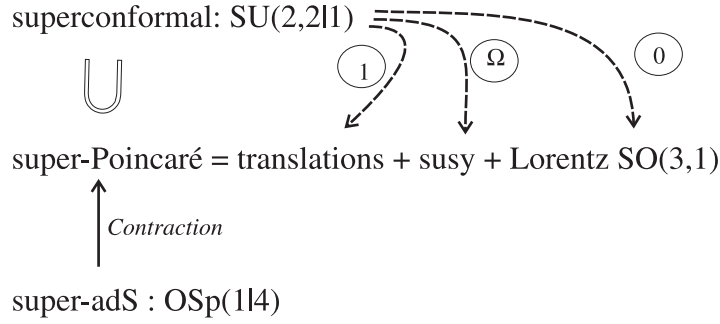


Figure 2. Schematic structure of superalgebras for $N = 1, d = 4$.

generators to P/x , and the supersymmetries Q to Q/\sqrt{x} , the superalgebra is⁴

$$\begin{aligned}
 \{Q, Q\} &= \gamma^\mu P_\mu + 2x\gamma^{\mu\nu} M_{\mu\nu}, & [P_\mu, Q] &= x\gamma_\mu Q, \\
 [M_{\mu\nu}, Q] &= -\frac{1}{4}\gamma_{\mu\nu} Q, & [P_\mu, P_\nu] &= 8x^2 M_{\mu\nu}, \\
 [P_\mu, M_{\nu\rho}] &= \eta_{\mu[\nu} P_{\rho]}, & [M_{\mu\nu}, M_{\rho\sigma}] &= \eta_{\mu[\rho} M_{\sigma]\nu} - \eta_{\nu[\rho} M_{\sigma]\mu}.
 \end{aligned} \tag{2.7}$$

The super-Poincaré theory is the contraction $x \rightarrow 0$ of this algebra.

When we consider the superconformal algebras, the dilatational weights of the generators form a table

$$\begin{aligned}
 1 &: P_\mu \\
 \frac{1}{2} &: Q \\
 0 &: D, M_{\mu\nu}, R \\
 -\frac{1}{2} &: S \\
 -1 &: K_\mu,
 \end{aligned} \tag{2.8}$$

where R is for this case $U(1)$, and in general can be a larger ‘ R -symmetry’ algebra. The super-Poincaré subalgebra is obtained from the positive weight operators, and the Lorentz part of the weight 0 operators. The others can appear nonlinearly.

There is an enlargement of the picture, see figure 3. Indeed, the two-index operator that appears on the right-hand side of the supersymmetry anticommutators, should not necessarily be identified with the Lorentz generators. We can give it the name $Z_{\mu\nu}$, suggesting its identification as a ‘central’ charge:

$$\{Q, Q\} = \gamma^\mu P_\mu + 2\gamma^{\mu\nu} Z_{\mu\nu}. \tag{2.9}$$

The Lorentz generators $M_{\mu\nu}$ are then an extra part of the algebra. Thus, there is a semi-direct sum of the Lorentz algebra and the algebra $OSp(1|4)$. Now we apply a different type of contraction. First, we also rescale $Z_{\mu\nu}$ in the same way as P_μ (remember that $M_{\mu\nu}$ was not

⁴ We omit spinor indices, as well as the charge-conjugation matrix C^{-1} that should be multiplied by the gamma matrices on the right-hand side of all anticommutation relations. This charge conjugation can be seen as the metric that lowers the (unwritten) spinor indices.

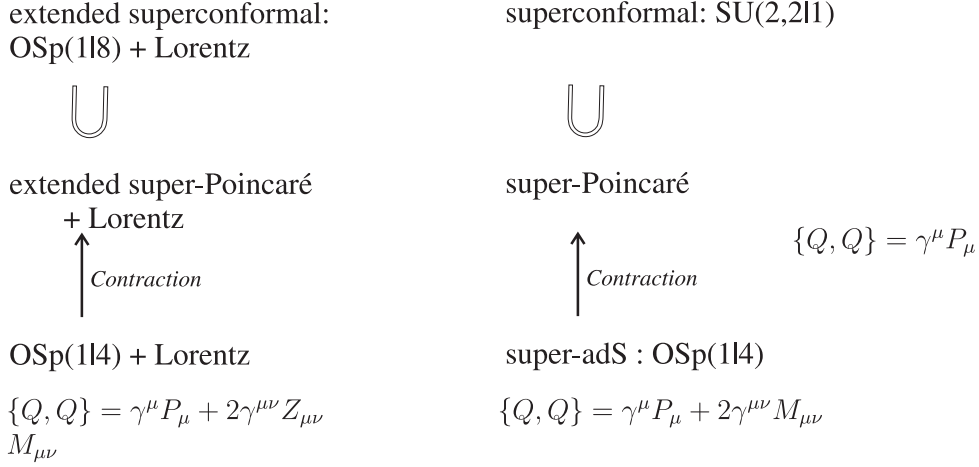


Figure 3. (Extended) algebras for $N = 1$, $d = 4$.

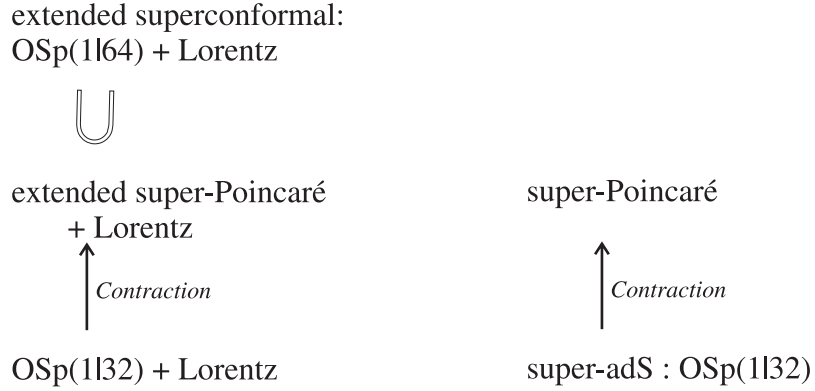


Figure 4. (Extended) algebras for $d = 11$.

rescaled), arriving at

$$\begin{aligned}
 \{Q, Q\} &= \gamma^\mu P_\mu + 2\gamma^{\mu\nu} Z_{\mu\nu}, & [P_\mu, Q] &= x\gamma_\mu Q, \\
 [Z_{\mu\nu}, Q] &= -\frac{1}{4}x\gamma_{\mu\nu} Q, & [P_\mu, P_\nu] &= 8x^2 Z_{\mu\nu}, \\
 [P_\mu, Z_{\nu\rho}] &= x\eta_{\mu[\nu} P_{\rho]}, & [Z_{\mu\nu}, Z_{\rho\sigma}] &= x\eta_{\mu[\rho} Z_{\sigma]\nu} - x\eta_{\nu[\rho} Z_{\sigma]\mu}.
 \end{aligned} \tag{2.10}$$

The rescaling in this case keeps the Z in the anticommutator of the supersymmetries, as in (2.9). The extended super-Poincaré algebra thus also contains the central charge, and the Lorentz generators as an extra part. This is a subalgebra of the extended superconformal algebra $\text{OSp}(1|8)$, which was already mentioned in [2] as a second possible $N = 1$, $d = 4$ superconformal algebra.

A similar picture can be made, for example, in $d = 11$ (see figure 4). The adS algebra that forms the basis is then $\text{OSp}(1|32)$. The anticommutator of the supersymmetries contains, apart from the translations, a two-index and a five-index generator:

$$\{Q_\alpha, Q_\beta\} = \Gamma_{\alpha\beta}^\mu P_\mu + 2\Gamma_{\alpha\beta}^{\mu\nu} Z_{\mu\nu} + \frac{1}{5!} \Gamma_{\alpha\beta}^{\mu\nu\rho\sigma\tau} Z_{\mu\nu\rho\sigma\tau}^5. \tag{2.11}$$

One may use uniform rescalings for all the bosonic generators (dividing them by x) and the fermionic Q (dividing them by \sqrt{x}). Then the anticommutator of the supersymmetries is not affected, but all the other commutators ([boson, boson] and [fermion, boson]) gain a factor of x on the right-hand side, and thus vanish in the corresponding Poincaré contracted theory. One may also identify the two-index operator as the Lorentz generator, as it has the appropriate commutation relations with the supersymmetries and with the bosonic generators. If this one is not rescaled by a factor of x , the contracted theory does not contain $M_{\mu\nu}$ on the right-hand side of the anticommutator of supersymmetries, and the appropriate commutators with the Lorentz generators do survive the limit. In this case, there is, however, only one superconformal algebra [2]. The positive-weight operators of that algebra do contain a $Z_{\mu\nu}$, and thus it is the extended super-Poincaré algebra that is a subgroup of this superconformal algebra.

Below, we will be considering this extended super-Poincaré algebra, having as generators those of $OSp(1|32)$. It is clear from the above that we could also consider it as a subalgebra of $OSp(1|64)$. The latter also contains the generators of negative weight that are important for the nonlinear realizations as in [4]. This algebra would also allow us to make connections to higher dimensions [6, 10]. However, the extra generators are not relevant for the issues that we treat here. Only the anticommutator of 32 supersymmetries, the only non-trivial one in the super-Poincaré theory, will be mentioned.

3. Complex symmetry algebras

$OSp(1|32)$ is the algebra of 32 fermionic charges with all possible bosonic generators in their anticommutator. The contraction, in the sense indicated in section 2, underlies the F-theory of 12 dimensions, the M-theory of 11 dimensions, and the IIA and IIB string theories in 10 dimensions. They are obtained by identifying appropriate subgroups of $Sp(32)$ as the Lorentz rotations. Note that in the case of the extended algebras of section 2, this $Sp(32)$ is the automorphism algebra of the supersymmetries, in the semi-direct product with $OSp(1|32)$. In any case, the supersymmetries should be in a spinor representation of the Lorentz group. Dimensional reduction and T-dualities are then obtained as mappings between generators of $OSp(1|32)$.

To recognize $OSp(1|32)$ as a symmetry algebra in d dimensions, one has to embed $SO(d)$ in $Sp(32)$, in such a way that the spinor representation of $SO(d)$ fits in the 32. This already makes clear that $d = 12$ is the highest possible dimension. To make that identification, we have to select chiral spinors \hat{Q} of 12 dimensions. These are defined using the chiral projection $\hat{\mathcal{P}}^+$:

$$\hat{\mathcal{P}}^+ \hat{Q} = \hat{Q}, \quad \hat{\mathcal{P}}^+ = \frac{1}{2}(1 + \hat{\Gamma}_*), \quad \hat{\Gamma}_* = \Gamma_1 \Gamma_2 \dots \Gamma_{12}. \quad (3.1)$$

note that we use the notation Γ_* (the hat specifies the 12-dimensional context) in any even dimension to denote the product of all the gamma matrices, similar to γ_5 in four dimensions. The anticommutator of the supersymmetries looks like

$$\{\hat{Q}, \hat{Q}\} = \frac{1}{2} \hat{\mathcal{P}}^+ \hat{\Gamma}^{\hat{M}\hat{N}} \hat{Z}_{\hat{M}\hat{N}} + \frac{1}{6!} \hat{\mathcal{P}}^+ \hat{\Gamma}^{\hat{M}_1 \dots \hat{M}_6} \hat{Z}_{\hat{M}_1 \dots \hat{M}_6}^+. \quad (3.2)$$

In 11 dimensions, the bosonic generators split as $528 = 11 + 55 + 462$, following the anticommutator (2.11). In 10 dimensions one can again define chiral spinors, which are 16-dimensional, and consider either two generators of opposite chirality (IIA) or of the same

chirality (IIB). In the first case, the anticommutators are

$$\begin{aligned}\{Q^\pm, Q^\pm\} &= \mathcal{P}^\pm \Gamma^M Z_M^\pm + \frac{1}{5!} \mathcal{P}^\pm \Gamma^{M_1 \dots M_5} Z_{M_1 \dots M_5}^\pm, \\ \{Q^\pm, Q^\mp\} &= \pm \mathcal{P}^\pm Z + \frac{1}{2} \mathcal{P}^\pm \Gamma^{MN} Z_{MN} \pm \frac{1}{4!} \mathcal{P}^\pm \Gamma^{M_1 \dots M_4} Z_{M_1 \dots M_4}.\end{aligned}\quad (3.3)$$

The 528 generators are thus split as $2 \times (10 + 126)$ in the anticommutators between generators of the same chirality and $1 + 45 + 210$ in the anticommutator between generators of opposite chirality.

For the IIB case, we have a doublet of fermionic generators Q^i ($i = 1, 2$), of the same chirality, and the anticommutators are

$$\begin{aligned}\{Q^i, Q^j\} &= \mathcal{P}^+ \Gamma^M Y_M^{ij} + \frac{1}{3!} \mathcal{P}^+ \Gamma^{MNP} \varepsilon^{ij} Y_{MNP} + \mathcal{P}^+ \frac{1}{5!} \Gamma^{M_1 \dots M_5} Y_{M_1 \dots M_5}^{ij}, \\ Y_M^{ij} &= \delta^{ij} Y_M^{(0)} + \tau_1^{ij} Y_M^{(1)} + \tau_3^{ij} Y_M^{(3)},\end{aligned}\quad (3.4)$$

where in the second line we have split the symmetric matrix Y^{ij} in three components, as we can also do for the five-index generators. Here the decomposition is $528 = (3 \times 10) + 120 + (3 \times 126)$.

It is clear that all of these algebras are related. The dimensional reductions relate the generators as follows. The chiral generator \hat{Q} of 12 dimensions, splits in 10 dimensions into a chiral and an antichiral generator, as follows from the relation $\hat{\Gamma}_* = \Gamma_* \otimes \sigma_3$ for a convenient realization of gamma matrices, where Γ_* is the product of 10 gamma matrices of dimension 32×32 in 10 dimensions (for the realization that we use in any dimension see [16]). The two chiral generators are the Q^\pm in (3.3), and adding them gives the 32-component generator $Q = Q^+ + Q^-$ used for $d = 11$. The T-dual theories are identified by taking

$$Q^+ = Q^1, \quad Q^- = \Gamma^s Q^2, \quad (3.5)$$

where Γ^s is a gamma matrix in an arbitrary (spacelike or timelike) direction. On the other hand, S-duality is the mapping

$$Q^i \xrightarrow{S} \left(e^{i \frac{1}{4} \pi \tau_2} \right)^i_j Q^j. \quad (3.6)$$

Thus all the dimensional reductions and dualities are written as mappings between the generators of $OSp(1|32)$. We mentioned here only the fermionic generators explicitly, as the rules for the bosonic generators follow from identifying the anticommutator relations before and after the map.

4. Real symmetry algebras

The important fact for the real forms is the uniqueness of the real form of the superalgebra $OSp(1|32)$. Therefore, the equivalences of all the symmetry algebras of section 3 are also valid for the real form, when it exists. The real form exists only for specific spacetime signatures. The dimensional reduction and T-duality now acts between theories of specific signatures. We then have to distinguish space/space, time/time and space/time dualities.

In general, a complex algebra has different real forms. Even for an algebra as small as $SU(2)$,

$$[T_1, T_2] = T_3, \quad [T_2, T_3] = T_1, \quad [T_3, T_1] = T_2, \quad (4.1)$$

there are already two inequivalent ‘real forms’. Either one considers the set $a^1 T_1 + a^2 T_2 + a^3 T_3$ with $a^i \in \mathbb{R}$, which is the real form $SU(2)$, or one can consider $ib^1 T_1 + ib^2 T_2 + b^3 T_3 = b^i S_i$,

Table 1. The possible spacetime signatures for 32 real spinor generators. The first column indicates the number of complex generators that are present before any projection. The last row indicates, for each signature, whether in $d = 10$ a real form for type IIA (A), type IIB (B) or both (A/B) exists.

12		(10, 2)				(6, 6)
64		MW				MW
11	(10, 1)		(9, 2)			(6, 5) = (5, 6)
32	M		M			M
10	(10, 0)	(9, 1)	(8, 2)	(7, 3)	(6, 4)	(5, 5)
32	SM	MW	M	SMW	SM	MW
	A	A/B	A	B	A	A/B

with $b^i \in \mathbb{R}$, for $S_1 = iT_1$, $S_2 = iT_2$ and $S_3 = T_3$. Another way of saying this is that either the T or the S generators can be considered as real. In both cases i does not appear in the commutation relations, see

$$[S_1, S_2] = -S_3, \quad [S_2, S_3] = S_1, \quad [S_3, S_1] = S_2, \quad (4.2)$$

which differs by one minus sign from (4.1). The minus sign cannot be eliminated by real redefinitions. The fact that they are the same complex algebra means that they are the same by complex redefinitions.

Considering the table of real forms of the basic Lie superalgebras ‘of classical type’ [17] (see, e.g., table 5 of [16] for a convenient presentation), we see that nearly all superalgebras have different real forms, even the exceptional superalgebras. However, the algebras $OSp(1|n)$ have only one real form, with $Sp(n, \mathbb{R})$ as bosonic subalgebra.

To realize this real algebra in d dimensions, we have to consider when we can impose consistent reality conditions on the fermionic generators. This is sufficient to be able to classify all the realizations of the unique real superalgebra $OSp(1|32)$. This has been investigated in [18], and table 2 of [16] gives the summary of the results that we need. We need 32 real supercharges. The table shows that $d = 12$ with (space, time) signature (10, 2) is the highest possible dimension. In general the results are invariant under $(s, t) \simeq (s - 4, t + 4)$, thus (6, 6) is also possible. The interchange of s and t is irrelevant, and corresponds merely to a change of notation of mostly + to mostly – metrics. Therefore, we do not mention the (2, 10) signature. To make the projections to real spinors one uses three types of projections, Weyl, Majorana or symplectic Majorana. This leads to the possibilities for 32-component real spinors listed in table 1.

One can then consider the dimensional reductions and T-dualities discussed in section 3, but now we have to be careful with the signatures. When performing the T-duality as in (3.5), one has to distinguish whether Γ^s is a timelike or a spacelike gamma matrix. This s -direction can even be timelike for the IIA algebra and spacelike for the IIB algebra or vice versa. These are the time/space or space/time T-dualities, changing the signature. This leads to the both-sided arrows in figure in table 2 of [14].

5. Translations and the energy operator

In the third step we identify one of the vector generators as ‘translations’. This identification is essential for a spacetime interpretation of the theory. The different possibilities for this identification distinguish, for example, IIA from IIA* theories. We will then note that T-duality gives a mapping between different types of generators. It can mix, for example, translations

with ‘central charges’. Finally, we will see that there is a unique positive-energy operator in the algebra. However, that generator is not always the timelike component of the translations. For instance, in IIA theories, the positive operator is P_0 , but in IIA* theories it is another one, and thus P_0 is not positive in that case.

So far, all bosonic generators were treated on an equal footing. To make the connection between algebras and a spacetime theory, we want to know which generator performs ‘translations’ in spacetime. Seen in another way, the spacetime is the manifold defined from a base point by the action of this ‘translation’ generator. This is thus similar to the coset space idea. To generate a spacetime of the appropriate dimension, the translation operator should be a vector operator in the theory. This is nearly the only requirement, apart from a non-degeneracy condition. Indeed, in order for the supersymmetries to perform their usual role, their square should give the translations. Thus the matrix that appears in the anticommutator between all the supersymmetries, defining how they square to give translations, should be non-degenerate.

For $d = 12$, with the algebra (3.2), there is no vector operator. Thus there is no candidate for translations, implying that F-theory has no straightforward spacetime interpretation. On the other hand, for $d = 11$, with the algebra (2.11), there is one vector operator, and this one should thus be called the translation generator.

In 10 dimensions it becomes more interesting. Consider first the IIA algebra (3.3). There are two vector operators Z_M^+ and Z_M^- . Both separately are not convenient, because then one-half of the supersymmetries would not square to give translations. However, we can take linear combinations. For the signature (9, 1) there are, up to redefinitions, two choices consistent with the reality conditions:

$$\begin{aligned} (9, 1): \quad \text{IIA} : \quad P_M &\equiv Z_M^+ + Z_M^-, \\ \text{IIA}^* : \quad P_M &\equiv Z_M^+ - Z_M^-. \end{aligned} \quad (5.1)$$

We label these choices IIA and IIA* in accordance with [11]. For signatures (10, 2) or (8, 2) there are the possibilities

$$\begin{aligned} (10, 0) \text{ or } (8, 2): \quad \text{IIA} : \quad P_M &\equiv i(Z_M^+ + Z_M^-), \\ \text{IIA}' : \quad P_M &\equiv Z_M^+ - Z_M^-. \end{aligned} \quad (5.2)$$

However, now these two choices can be related by a redefinition $Q^\pm \rightarrow e^{\pm i\pi/2} Q^\pm$. Such a redefinition is, similar to (3.6) and therefore we also recognize it as an S-duality.

The operators that are not translations, remain as ‘central charges’ in the theory. Therefore, we see that the generator that is a translation in one theory, appears as a central charge in the other theory, as we announced at the beginning of this section.

For the IIB algebra (3.4), there are three candidates for translations. For signature (9, 1) they are all consistent with the reality condition. We thus distinguish

$$\begin{aligned} (9, 1): \quad \text{IIB} : \quad P_M &= Y_M^{(0)}, \\ \text{IIB}^* : \quad P_M &= Y_M^{(3)}, \\ \text{IIB}' : \quad P_M &= Y_M^{(1)}. \end{aligned} \quad (5.3)$$

Considering the possible redefinitions, we come back to (3.6). This S-duality leaves the IIA translation generator invariant, and relates the translation of IIB* with that of IIB'. On the other hand, for the signature (7, 3) there are three S-dual versions.

Finally, we can identify one bosonic operator in $OSp(1|32)$ that is positive. We can identify this one from the anticommutation relation

$$\{Q^i, Q^{j*}\} \geq 0, \quad (5.4)$$

using the Majorana condition. Denoting

$$\{Q^i, Q^j\} = Z^{ij} C^{-1} \quad (5.5)$$

to represent all the anticommutation relations, where Z^{ij} is a matrix in spinor space as well as in the i, j indices, this implies

$$Z^{ij} \Gamma^i \dots \Gamma^j \geq 0, \quad (5.6)$$

exhibiting all the timelike Γ -matrices. Therefore, the trace of that operator has positive eigenvalues. When we split Z as usual into different irreducible representations for the spacetime Lorentz group, then, in order to absorb the gamma matrices, the relevant part of Z has as many spacetime indices as there are time directions. All of its directions should be timelike. Thus for Minkowski spaces it is the timelike component of a vector operator, while for Euclidean theories this positive operator is a scalar ‘central charge’. For Minkowski theories we can thus wonder whether the positive-energy operator is the timelike component of the operator that we selected as ‘translations’. If this is the case then the usual Hamiltonian will be positive. When the positive energy is the timelike component of another vector operator, then the Hamiltonian built from P_0 is not positive definite. This is what happens in the IIA*, IIB* and IIB’ theories. In these theories, the kinetic energies of some of the p -form gauge fields are negative definite. As an example consider the vector operators in the IIB-like theories, as in (3.4). With our convention that $\alpha = 1$, the trace in (5.6) selects the $M = 0$ component of $Y_M^{(0)}$ as the positive-definite energy. Thus it is indeed the IIB theory where the energy is the timelike part of translations, and not for the other versions. The algebraic approach thus gives an understanding of the positivity in type IIA and IIB versus the lack of positivity in the other theories.

6. Conclusions

The algebras of F-theory, M-theory, type IIA and IIB, etc are different faces of the same superalgebra $OSp(1|32)$. The uniqueness of the real form of that algebra implies that all of these manifestations can be related by mappings between the generators of the algebra. This holds especially for the dimensional reductions, T- and S-dualities that relate these theories. Different spacetime signatures are easily incorporated. However, for certain spacetime signatures, some theories may exist only in complex form. This answers the question of why we sometimes need a complexification procedure to obtain a Euclidean theory. In particular, the IIB theory has no real form in $(10, 0)$. Therefore, in order to discuss the D-instanton in IIB, we have to give up the concept of a theory with real fields and action.

We have understood the *-theories as being distinguished from the usual IIA and IIB theories by a different identification of the translation generator. They are related to the ordinary theories by a ‘duality’ interchanging translations with central charges. Thus in these dualities the concept of spacetime is very intriguing. It should be interchanged with a sort of harmonic space where coordinates are also associated with other (vector) central charges. The unique positive-energy operator is the timelike component of the translations in the ordinary type IIA and IIB theories, but in other versions (*-theories or theories with a different signature), it is not the P_0 operator that is positive, but rather a component of a central charge operator.

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